Tariffs, Trade, and Finance in a Three-Country Global Economy: A Differential Model

Julio Huato*

June 11, 2025

Abstract

A simplified, linear IS-LM-type general-equilibrium model of a three-country global economy (United States, China, rest of the world) helps to explore short-run shifts in the structure of international trade and finance induced by tariffs, such as those proposed by U.S. president Trump in 2025. Using gold as money and an endogenous global interest rate, the model captures interactions among income, consumption, investment, and trade, emphasizing global constraints to domestic policymaking in a multipolar world. Two versions are analyzed: one where tariffs reduce imports indirectly via disposable income, and another where tariffs directly lower imports linearly. A differential approach circumvents the Walras-law overdeterminacy of the level-based general-equilibrium framework, focusing on equilibrium shifts. Calibrated with real-world data, simulations compare a zero-tariff baseline to U.S. tariffs (100% on China, 10% on rest of the world) and those with Chinese retaliation (100% on U.S. imports). Results show that U.S. tariffs modestly reduce the trade deficit but erode income and consumption, shifting relative economic power toward China and the rest of the world. Retaliation mitigates China's losses, amplifying global spillovers. The model highlights the counterproductive effects and loss of global macro efficiency that results from tariffs.

*CUNY John Jay College of Criminal Justice, jhuato@jjay.cuny.edu. Personal email: juliohuato@gmail.com.

1 Preamble

This paper develops a general-equilibrium model of the IS-LM type (Hicks 1937 [1], 1980 [2]) as a pedagogical device for analyzing short-run shifts in the international structure of trade and finance triggered by tariff policies, such as those proposed by U.S. president Donald Trump in 2025. The model features a three-country global economy (United States, China, rest of the world) trading final goods and financial claims, with gold as the ultimate money. This abstraction highlights the global constraints facing policymakers in an emerging multipolar world, which avoids complex central bank currency dynamics.

The model offers a simplified framework to explore how tariffs affect income, consumption, investment, and trade between nations. By specifying all relationships linearly, the model is solvable with basic linear algebra. Two versions are considered: one where tariffs reduce imports indirectly by shrinking disposable income, and another where tariffs directly scale imports. To address the overdeterminacy of the level-based general-equilibrium framework implied by Walras' law, a differential form is derived, focusing on changes in endogenous variables (ΔY_i, Δr).[1] Calibrated to approximate real-world economic conditions (e.g., U.S. trade deficit 3% GDP, BEA 2024 [4]), simulations reveal tariffs' trade-offs, often backfiring by degrading U.S. economic power.

2 Overview

The model features three countries ($i = 1, 2, 3$), each with households, businesses, and government sectors operating in goods, primary-factor, and financial markets. International trade occurs in final goods and financial claims, while labor and capital services are exchanged domestically. Gold serves as money, abstracting from central-bank currency dynamics and emphasizing global financial integration. The global (gold) interest rate balances an in-

[1] This paper follows up on Huato (2025, [3]) were the general-equilibrium model in the levels is introduced, calibrated, and solved numerically. In its level form, the model has an infinite number of solutions. For a unique solution, one of the endogenous variables can be parameterized and the remaining subsystem solved in terms of the parameterized variable. Alternatively (and this is the route followed in that paper), a numerical solution can be obtained with the calibration of the model serving to anchor the solution to a local equilibrium.

tegrated global financial market, reflecting worldwide borrowing and lending pressures.

Two versions of the model specify tariffs τ_{ij} (barriers by country i on imports from country j) differently:

- Version 1: Tariffs act as taxes, reducing disposable income and indirectly lowering imports via fixed import propensities.
- Version 2: Tariffs directly scale imports via $X_{ij} = \alpha_i q_{ij} Y_j \left(\frac{1}{1+\tau_{ij}}\right)$, modeling direct trade volume reductions.

By Walras' law, the level-based general-equilibrium model is overdetermined.[2] A differential approach resolves this by solving for changes in the endogenous variables (ΔY_i, Δr), capturing equilibrium shifts triggered by tariff changes. Simulations, calibrated with data from BEA (2024 [4]), IMF (2024 [5]), World Bank (2025 [6], 2025a [7]), and OECD (2024 [8]), analyze tariff scenarios, highlighting trade-offs in economic policy.

3 Model

Each country's economy is owned by households, with government and businesses as their agencies. The base model is specified in levels but solved in differentials.

Consumption follows the rule:

$$C_i = \bar{C}_i + c_i(Y_i - T_i), \tag{1}$$

where $\bar{C}_i$ is base consumption, c_i is the marginal propensity to consume, and T_i is taxes.

Investment is:

$$I_i = \bar{I}_i - dr, \tag{2}$$

where $\bar{I}_i$ is base investment, d is interest rate sensitivity, and r is the global interest rate.

Government spending is exogenous:

$$G_i = \bar{G}_i. \tag{3}$$

[2] Walras' law implies that equilibrium in $n-1$ markets (e.g., goods markets) ensures equilibrium in the n-th market (e.g., global financial market).

The government budget constraint is:

$$T_i + B_i^g = G_i, \tag{4}$$

where B_i^g is government borrowing (e.g., placement of bonds, currency, social-insurance policies, etc.).

Household savings are:

$$S_i = Y_i - T_i - C_i. \tag{5}$$

Financial market equilibrium balances savings and borrowing:

$$S_i = B_i^b + B_i^g + B_i^a, \tag{6}$$

$$B_i^b = I_i, \tag{7}$$

$$B_i^a = \sum_{j \neq i} X_{ij} - \sum_{j \neq i} X_{ji}. \tag{8}$$

The global balance of payments is:

$$\sum_i B_i^a = 0, \quad \sum_i S_i = \sum_i (B_i^g + B_i^b + B_i^a). \tag{9}$$

In the first version of the model, tariffs are specified as taxes.

Exports (imports from j to i) are:

$$X_{ij} = \alpha_i q_{ij} Y_j, \tag{10}$$

where α_i is export efficiency and q_{ij} is j's propensity to import from i.

Taxes include tariff revenue:

$$T_i = t_i Y_i + \sum_{j \neq i} \tau_{ij}. \tag{11}$$

Income-expenditure equilibrium is:

$$Y_i = C_i + I_i + G_i + \sum_{j \neq i} \alpha_i q_{ij} Y_j - \sum_{j \neq i} \alpha_j q_{ji} Y_i. \tag{12}$$

Define m_i:

$$m_i \equiv 1 - c_i(1 - t_i) + \sum_{j \neq i} \alpha_j q_{ji}, \tag{13}$$

and combine equations:

$$m_iY_i = \bar{C}_i + \bar{I}_i - dr + \bar{G}_i - c_i \sum_{j \neq i} \tau_{ij} + \sum_{j \neq i} \alpha_i q_{ij} Y_j. \tag{14}$$

Differentiate:

$$m_i \Delta Y_i = -d\Delta r - c_i \sum_{j \neq i} \Delta \tau_{ij} + \sum_{j \neq i} \alpha_i q_{ij} \Delta Y_j. \tag{15}$$

The global financial balance differentiation yields:

$$\sum_i k_i \Delta Y_i + 3d\Delta r = -\sum_i c_i \sum_{j \neq i} \Delta \tau_{ij}, \tag{16}$$

where $k_i \equiv 1 - c_i + c_i t_i + t_i$. Appendix B shows the step-by-step derivation.

In the second version of the model, tariffs reduce imports directly.

Exports are:

$$X_{ij} = \alpha_i q_{ij} Y_j \left(\frac{1}{1 + \tau_{ij}} \right). \tag{17}$$

Taxes are:

$$T_i = t_i Y_i + \sum_{j \neq i} \tau_{ji} \alpha_j q_{ji} Y_i \left(\frac{1}{1 + \tau_{ji}} \right). \tag{18}$$

Income-expenditure equilibrium is:

$$Y_i = C_i + I_i + G_i + \sum_{j \neq i} \alpha_i q_{ij} Y_j \left(\frac{1}{1 + \tau_{ij}} \right) - \sum_{j \neq i} \alpha_j q_{ji} Y_i \left(\frac{1}{1 + \tau_{ji}} \right). \tag{19}$$

Redefine m_i:

$$m_i \equiv 1 - c_i \left(1 - t_i - \sum_{j \neq i} \frac{\tau_{ji} \alpha_j q_{ji}}{1 + \tau_{ji}} \right) + \sum_{j \neq i} \frac{\alpha_j q_{ji}}{1 + \tau_{ji}}, \tag{20}$$

and combine:

$$m_i Y_i = \bar{C}_i + \bar{I}_i - dr + \bar{G}_i + \sum_{j \neq i} \frac{\alpha_i q_{ij} Y_j}{1 + \tau_{ij}}. \tag{21}$$

Differentiate:

$$m_i \Delta Y_i + Y_i \sum_{j \neq i} \alpha_j q_{ji} \cdot \frac{1 - c_i}{(1 + \tau_{ji})^2} \Delta \tau_{ji} =$$

$$-d\Delta r + \sum_{j \neq i} \frac{\alpha_i q_{ij} \Delta Y_j}{1 + \tau_{ij}} - \sum_{j \neq i} \frac{\alpha_i q_{ij} Y_j \Delta \tau_{ij}}{(1 + \tau_{ij})^2}. \tag{22}$$

The global financial balance differentiation is:

$$\sum_i \left(1 - c_i + c_i t_i + c_i \sum_{j \neq i} \frac{\tau_{ji} \alpha_j q_{ji}}{1 + \tau_{ji}}\right) \Delta Y_i + 3d\Delta r =$$

$$-\sum_i c_i \sum_{j \neq i} \frac{\alpha_j q_{ji} Y_i \Delta \tau_{ji}}{(1 + \tau_{ji})^2}. \tag{23}$$

See Appendix B for the step-by-step derivation.

The differential systems are solved numerically using the R code in Appendix C, calibrated to approximate the United States ($i = 1$), China ($i = 2$), and rest of the world ($i = 3$). Parameters are:

- $\bar{C}_i = [2, 1, 3]$ (base consumption, 10-15% GDP, BEA 2024 [9]),
- $c_i = [0.8, 0.6, 0.7]$ (savings rates, World Bank 2025 [6]),
- $t_i = [0.25, 0.15, 0.2]$ (tax rates, OECD 2024 [8]),
- $\bar{I}_i = [5, 5, 8]$ (investment 19-33% GDP, BEA 2024 [9]),
- $\bar{G}_i = [5, 4, 9]$ (government spending, IMF 2024 [5]),
- $\alpha_i = [1, 1, 1]$ (trade efficiency, World Bank 2025 [7]),
- $d = 2$ (tuned for positive investment),
- $q_{ij} = \begin{bmatrix} 0 & 0.033 & 0.0375 \\ 0.02 & 0 & 0.0875 \\ 0.08 & 0.133 & 0 \end{bmatrix}$ (trade shares, BEA 2024 [4], UN Comtrade 2024 [10]),
- Version 2 baseline GDP: $Y_i = [23.39, 17.56, 38.71]$ (approximating BEA 2024 [9]: \$28.65T, IMF 2024 [5]: \$18T, World Bank 2025 [6]).

Simulations compare a baseline ($\Delta \tau_{ij} = 0$) against (1) U.S. tariffs ($\Delta \tau_{12} = 1$, $\Delta \tau_{13} = 0.1$) and (2) U.S. tariffs with Chinese retaliation ($\Delta \tau_{21} = 1$).

4 Results

The results are summarized in Tables 1, 2, and 3 in Appendix A, showing changes in GDP (ΔY_i), trade balances (ΔB_i^a), interest rates (Δr), and GDP shares across scenarios.

In version 1 (tariffs as taxes), U.S. unilateral tariffs reduce U.S. GDP ($\Delta Y_1 = -0.9214$) while boosting China ($\Delta Y_2 = 0.6311$) and the rest of the world ($\Delta Y_3 = 0.6784$). The U.S. trade deficit shrinks ($\Delta B_1^a = 0.1384$), but consumption falls sharply ($\Delta C_1 = -1.4328$) as tariffs pinch household budgets. The interest rate drops ($\Delta r = -0.1865$), spurring investment ($\Delta I_i = 0.3731$). With Chinese retaliation, U.S. GDP loss moderates ($\Delta Y_1 = -0.6502$), China's GDP dips slightly ($\Delta Y_2 = -0.0240$), and the rest of the world expands further ($\Delta Y_3 = 0.8307$). The U.S. deficit reduction weakens ($\Delta B_1^a = 0.0954$), with deeper consumption losses ($\Delta C_1 = -1.2701$).

In version 2 (linear tariff scaling of imports), U.S. tariffs cause GDP declines across all countries, most severely in the U.S. ($\Delta Y_i = [-1.3882, -0.3516, -0.2643]$). The U.S. trade deficit worsens ($\Delta B_1^a = -0.6073$) due to sharp export drops ($\Delta X_{12} = -0.5911$). Consumption falls ($\Delta C_1 = -1.5206$), and the interest rate rises ($\Delta r = 0.0260$), curbing investment ($\Delta I_i = -0.0521$). With Chinese retaliation, GDP losses deepen ($\Delta Y_i = [-1.5051, -0.9875, -0.3271]$), but the U.S. deficit improves slightly ($\Delta B_1^a = -0.1512$), with further consumption and investment declines ($\Delta C_1 = -1.5907$, $\Delta I_i = -0.1105$).

The analysis of country shares (Table 3) shows the U.S. share of global GDP (baseline: 29.36%) dropping to 28.62% under unilateral tariffs in version 1, with China (22.78%) and the rest of the world (48.60%) gaining. Retaliation raises the U.S. share slightly to 28.83%, but the rest of the world benefits most (48.76%). In version 2, the U.S. share falls to 28.47% (unilateral) and 28.37% (retaliation), with China and the rest of the world gaining. These shifts highlight tariffs' erosion of U.S. economic dominance, with global spillovers favoring competitors.

Version 1 suggests tariffs reduce the U.S. trade deficit at the cost of GDP and consumption losses, benefiting China and the rest of the world. Version 2's direct import reduction amplifies trade disruptions, causing universal GDP declines and financial tightening ($\Delta r > 0$). The linear import scaling in version 2 may overstate trade disruptions compared to empirical elasticities (World Bank 2025 [7]), but both versions underscore tariffs' counterproductive nature.

5 Conclusion

This differential model offers a robust and clear framework for analyzing tariff policies, sidestepping the overdeterminacy of the level-based general-equilibrium framework by focusing on equilibrium shifts. It highlights tariffs' trade-offs: modest deficit reductions at the expense of GDP, consumption, and relative economic power. Version 1 shows U.S. tariffs harming domestic consumers while boosting competitors, while version 2's direct import reduction amplifies global losses, potentially overstating trade disruptions due to its linear specification. The gold-based monetary system, by ignoring the dynamics of national currency monetary systems, may overemphasize gold's monetary role in the global economy. However, rising gold holdings (World Gold Council 2024 [11]) suggest its increased relevance as a hedge against currency volatility and de-facto remonetization.

Future extensions could incorporate nonlinear dynamics, explicit currency markets, or class-based income distributions to capture domestic and international conflicts. The model's simplicity, while a strength for teaching, limits its ability to capture complex real-world dynamics, such as trade elasticities or monetary policy responses. Nonetheless, it provides directional insights into tariffs' global impacts, aligning with the intuition that trade wars often backfire on those who initiate them.

References

[1] John R. Hicks. Mr. Keynes and the "Classics": A Suggested Interpretation. *Econometrica*, 5(5):147–159, April 1937.

[2] John R. Hicks. "IS-LM": An Explanation. *Journal of Post Keynesian Economics*, 3(2):139–154, Winter 1980-1981.

[3] Julio Huato. Tariffs, trade, and finance restructuring in a global economy with three countries. *Working Paper*, 2025.

[4] Bureau of Economic Analysis. U.s. international trade in goods and services, december and annual 2024, February 2025. Accessed April 11, 2025.

[5] International Monetary Fund. World economic outlook: Navigating global divergences. Technical report, International Monetary Fund, October 2024. Accessed April 11, 2025.

[6] World Bank. World development indicators: Gdp (current us$), 2025. Accessed April 11, 2025.

[7] World Bank. World integrated trade solution (wits): Trade statistics, 2025. Accessed April 11, 2025.

[8] Organisation for Economic Co-operation and Development. Revenue statistics 2024: Tax revenue trends in the oecd. Technical report, OECD, December 2024. Accessed April 11, 2025.

[9] Bureau of Economic Analysis. Gross domestic product, fourth quarter and annual 2024 (advance estimate), January 2025. Accessed April 11, 2025; projected release date.

[10] United Nations. Un comtrade database: International trade statistics, bilateral trade flows, 2024, 2025. Accessed April 11, 2025.

[11] World Gold Council. Gold demand trends: Full year 2024. Technical report, World Gold Council, February 2025. Accessed April 13, 2025.

Appendix A: Supplementary Tables

Table 1: Changes in economic variables (Version 1: Tariffs as taxes)

Scenario	ΔY_i	ΔB_i^a	Δr
Baseline	[0, 0, 0]	[0, 0, 0]	0
US tariffs	[-0.9214, 0.6311, 0.6784]	[0.1384, -0.0638, -0.0746]	-0.1865
With China's response	[-0.6502, -0.0240, 0.8307]	[0.0954, 0.0637, -0.1590]	-0.2623

Note: Values in trillions of gold units (≈ USD), except Δr. ΔY_i, ΔB_i^a listed as [USA, China, ROW].

Table 2: Changes in economic variables (Version 2: Linear tariff scaling)

Scenario	ΔY_i	ΔB_i^a	Δr
Baseline	[0, 0, 0]	[0, 0, 0]	0
US tariffs	[-1.3882, -0.3516, -0.2643]	[-0.6073, 0.5870, 0.0204]	0.0260
With China's response	[-1.5051, -0.9875, -0.3271]	[-0.1512, 0.2169, -0.0657]	0.0552

Note: Values in trillions of gold units (≈ USD), except Δr. ΔY_i, ΔB_i^a listed as [USA, China, ROW].

Table 3: Relative GDP shares (% of global GDP)

Scenario	USA	China	ROW
Baseline	29.36	22.04	48.60
Version 1: US tariffs	28.62	22.78	48.60
Version 1: With China's response	28.83	22.00	48.76
Version 2: US tariffs	28.47	21.84	48.69
Version 2: With China's response	28.37	21.43	48.76

Note: Shares computed using baseline GDP ($Y_i = [23.39, 17.56, 38.71]$) adjusted by ΔY_i. Global GDP is sum of $Y_i + \Delta Y_i$.

Appendix B: Derivations

This appendix derives the differential form of the global balance of payments (BoP) equation for both versions of the model, as referenced in equations 16 and 23.

For version 1, with tariffs as taxes, start with the global balance of payments (equation 9):

$$\sum_i S_i = \sum_i (B_i^g + B_i^b + B_i^a).$$

Substitute $B_i^g = G_i - T_i$, $B_i^b = I_i$, and $B_i^a = \sum_{j \neq i} X_{ij} - \sum_{j \neq i} X_{ji}$:

$$\sum_i S_i = \sum_i (G_i - T_i + I_i + \sum_{j \neq i} X_{ij} - \sum_{j \neq i} X_{ji}).$$

Since $\sum_i \sum_{j \neq i} X_{ij} = \sum_i \sum_{j \neq i} X_{ji}$ (global exports equal global imports), the balance of payments simplifies to:

$$\sum_i S_i = \sum_i (G_i - T_i + I_i).$$

Substitute $S_i = Y_i - T_i - C_i$, $C_i = \bar{C}_i + c_i(Y_i - T_i)$, $I_i = \bar{I}_i - dr$, $G_i = \bar{G}_i$, and $T_i = t_i Y_i + \sum_{j \neq i} \tau_{ij}$:

$$S_i = Y_i - t_i Y_i - \sum_{j \neq i} \tau_{ij} - \left[\bar{C}_i + c_i (Y_i - t_i Y_i - \sum_{j \neq i} \tau_{ij}) \right].$$

Simplify:

$$S_i = (1 - c_i)(1 - t_i) Y_i - \bar{C}_i - (1 - c_i) \sum_{j \neq i} \tau_{ij}.$$

The right-hand side is:

$$G_i - T_i + I_i = \bar{G}_i - (t_i Y_i + \sum_{j \neq i} \tau_{ij}) + (\bar{I}_i - dr).$$

Sum globally:

$$\sum_i S_i = \sum_i \left[(1 - c_i)(1 - t_i) Y_i - \bar{C}_i - (1 - c_i) \sum_{j \neq i} \tau_{ij} \right],$$

$$\sum_i (G_i - T_i + I_i) = \sum_i \left(\bar{G}_i - t_i Y_i - \sum_{j \neq i} \tau_{ij} + \bar{I}_i - dr \right).$$

Equate and simplify:

$$\sum_i \left[(1 - c_i)(1 - t_i) + t_i\right] Y_i + 3dr = \sum_i \left[\bar{C}_i + \bar{I}_i + \bar{G}_i - (1 - c_i) \sum_{j \neq i} \tau_{ij} \right].$$

Define $k_i = 1 - c_i + c_i t_i + t_i$. Differentiate, noting $\bar{C}_i$, $\bar{I}_i$, $\bar{G}_i$ are constants:

$$\sum_i k_i \Delta Y_i + 3d\Delta r = -\sum_i (1 - c_i) \sum_{j \neq i} \Delta \tau_{ij}.$$

Since $(1 - c_i) = -(c_i - 1)$, rewrite:

$$\sum_i k_i \Delta Y_i + 3d\Delta r = -\sum_i c_i \sum_{j \neq i} \Delta \tau_{ij},$$

yielding equation 16.

For version 2, with tariffs linearly scaling imports, use the same balance of payments and substitute

$$T_i = t_i Y_i + \sum_{j \neq i} \tau_{ji} \alpha_j q_{ji} Y_i \left(\frac{1}{1 + \tau_{ji}} \right),$$

,

$$X_{ij} = \alpha_i q_{ij} Y_j \left(\frac{1}{1 + \tau_{ij}} \right)$$

into the savings equation:

$$S_i = Y_i - \left[t_i Y_i + \sum_{j \neq i} \tau_{ji} \alpha_j q_{ji} Y_i \left(\frac{1}{1 + \tau_{ji}} \right) \right] -$$
$$\left[\bar{C}_i + c_i \left[Y_i - t_i Y_i - \sum_{j \neq i} \tau_{ji} \alpha_j q_{ji} Y_i \left(\frac{1}{1 + \tau_{ji}} \right) \right] \right].$$

Expand and then simplify:

$$S_i = (1 - c_i) \left(1 - t_i - \sum_{j \neq i} \frac{\tau_{ji} \alpha_j q_{ji}}{1 + \tau_{ji}} \right) Y_i - \bar{C}_i.$$

The right-hand side remains:

$$G_i - T_i + I_i = \bar{G}_i - \left[t_i Y_i + \sum_{j \neq i} \tau_{ji} \alpha_j q_{ji} Y_i \left(\frac{1}{1 + \tau_{ji}} \right) \right] + (\bar{I}_i - dr).$$

Sum and equate, then differentiate:

$$\sum_i \left(1 - c_i + c_i t_i + c_i \sum_{j \neq i} \frac{\tau_{ji} \alpha_j q_{ji}}{1 + \tau_{ji}} \right) \Delta Y_i + 3d\Delta r = -\sum_i c_i \sum_{j \neq i} \frac{\alpha_j q_{ji} Y_i \Delta \tau_{ji}}{(1 + \tau_{ji})^2},$$

which yields equation 23. The right-hand side accounts for tariff changes affecting tax revenue and consumption.

Appendix C: R Code

```
# Differential model simulations for trade and finance in a 3-country global economy
# Versions 1 (tariffs as taxes) and 2 (linear tariff scaling of imports)
# 4/17/2025

library(nleqslv)
library(knitr)

# Define parameters (calibrated to approximate real-world data)
params <- list(
  c = c(0.8, 0.6, 0.7),                    # Marginal propensity to consume (BEA, World Bank)
  t = c(0.25, 0.15, 0.2),                  # Income tax rates (OECD)
  alpha = c(1, 1, 1),                      # Export efficiency (World Bank)
  q = matrix(c(                            # Import propensities (BEA, UN Comtrade)
    0, 0.033, 0.0375,
    0.02, 0, 0.0875,
    0.08, 0.133, 0
  ), nrow = 3, byrow = TRUE),
  d = 2,                                   # Investment sensitivity to interest rate
  Y = c(23.39, 17.56, 38.71)              # Baseline GDP for version 2 (BEA, IMF, World Bank)
)

# Baseline tariffs (zero)
tau_baseline <- matrix(0, nrow = 3, ncol = 3)

# Version 1: Tariffs reduce disposable income
solve_diff_version1 <- function(params, delta_tau) {
  n <- 3
  m <- numeric(3)
  for (i in 1:3) {
    m[i] <- 1 - params$c[i] * (1 - params$t[i]) + sum(params$alpha[-i] * params$q[-i, i])
  }

  solve_system <- function(z) {
    dY <- z[1:n]
    dr <- z[n+1]
    eq <- numeric(n+1)
    for (i in 1:n) {
      eq[i] <- m[i] * dY[i] - sum(params$alpha[i] * params$q[i, -i] * dY[-i]) +
               params$d * dr + params$c[i] * sum(delta_tau[i, -i])
    }
    eq[n+1] <- sum((1 - params$c + params$c * params$t + params$t) * dY) +
               3 * params$d * dr + sum(params$c * rowSums(delta_tau))
    eq
  }

  z_init <- c(0, 0, 0, 0)
  result <- nleqslv(z_init, solve_system, control = list(maxit = 1000))

  if (result$termcd != 1) {
    warning("Version 1: nleqslv did not converge: ", result$message)
  }

  dY <- result$x[1:n]
  dr <- result$x[n+1]
  dC <- params$c * (1 - params$t) * dY - params$c * rowSums(delta_tau)
  dT <- params$t * dY + rowSums(delta_tau)
  dI <- rep(-params$d * dr, n)
  dX <- matrix(0, n, n)
  for (i in 1:n) for (j in 1:n) dX[i, j] <- params$alpha[i] * params$q[i, j] * dY[j]
  dB_a <- rowSums(dX) - colSums(dX)

  return(list(dY = dY, dr = dr, dC = dC, dT = dT, dI = dI, dX = dX, dB_a = dB_a))
}

# Version 2: Tariffs scale imports linearly
solve_diff_version2 <- function(params, delta_tau, tau_base) {
  n <- 3
  m <- numeric(3)
  for (i in 1:3) {
    m[i] <- 1 - params$c[i] + params$c[i] * params$t[i] +
```

```
          sum(params$alpha[-i] * params$q[-i, i] * (1 + params$c[i] * tau_base[-i, i]) / (1 + tau_base[-i, i]))
  }

  solve_system <- function(z) {
    dY <- z[1:n]
    dr <- z[n+1]
    eq <- numeric(n+1)
    for (i in 1:n) {
      dm_i <- sum(params$alpha[-i] * params$q[-i, i] * (1 - params$c[i]) / (1 + tau_base[-i, i])^2 * delta_tau[-i, i])
      eq[i] <- m[i] * dY[i] + params$Y[i] * dm_i -
               sum(params$alpha[i] * params$q[i, -i] * (1 / (1 + tau_base[i, -i])) * dY[-i]) +
               sum(params$alpha[i] * params$q[i, -i] * params$Y[-i] * (1 / (1 + tau_base[i, -i])^2) * delta_tau[i, -i]) -
               params$d * dr
    }
    eq[n+1] <- sum((1 - params$c + params$c * params$t +
                    params$c * rowSums(tau_base * params$alpha * params$q / (1 + tau_base))) * dY) +
               3 * params$d * dr +
               sum(params$c * rowSums(params$alpha * params$q * params$Y / (1 + tau_base)^2 * delta_tau))
    eq
  }

  z_init <- c(0, 0, 0, 0)
  result <- nleqslv(z_init, solve_system, control = list(maxit = 1000))

  if (result$termcd != 1) {
    warning("Version 2: nleqslv did not converge: ", result$message)
  }

  dY <- result$x[1:n]
  dr <- result$x[n+1]
  dT <- (params$t + rowSums(tau_base * params$alpha * params$q / (1 + tau_base))) * dY +
        rowSums(params$alpha * params$q * params$Y / (1 + tau_base)^2 * delta_tau)
  dC <- params$c * (dY - dT)
  dI <- rep(-params$d * dr, n)
  dX <- matrix(0, n, n)
  for (i in 1:n) for (j in 1:n) {
    dX[i, j] <- params$alpha[i] * params$q[i, j] * (1 / (1 + tau_base[i, j])) * dY[j] -
                params$alpha[i] * params$q[i, j] * params$Y[j] * (1 / (1 + tau_base[i, j])^2) * delta_tau[i, j]
  }
  dB_a <- rowSums(dX) - colSums(dX)

  return(list(dY = dY, dr = dr, dC = dC, dT = dT, dI = dI, dX = dX, dB_a = dB_a))
}

# Define scenarios
scenarios <- list(
  baseline = matrix(0, nrow = 3, ncol = 3),
  us_tariffs = matrix(c(0, 1, 0.1, 0, 0, 0, 0, 0, 0), nrow = 3, byrow = TRUE),
  us_tariffs_retaliation = matrix(c(0, 1, 0.1, 1, 0, 0, 0, 0, 0), nrow = 3, byrow = TRUE)
)

# Run simulations
results <- list()
for (version in c("version1", "version2")) {
  results[[version]] <- lapply(names(scenarios), function(scenario) {
    delta_tau <- scenarios[[scenario]]
    if (version == "version1") {
      result <- solve_diff_version1(params, delta_tau)
    } else {
      result <- solve_diff_version2(params, delta_tau, tau_baseline)
    }
    result$scenario <- scenario
    result$version <- version
    return(result)
  })
}

# Format results for tables
for (version in c("version1", "version2")) {
  summary_table <- data.frame(
    Scenario = sapply(results[[version]], function(x) x$scenario),
    dY_USA = sapply(results[[version]], function(x) round(x$dY[1], 4)),
    dY_China = sapply(results[[version]], function(x) round(x$dY[2], 4)),
    dY_ROW = sapply(results[[version]], function(x) round(x$dY[3], 4)),
    dr = sapply(results[[version]], function(x) round(x$dr, 4)),
```

```
    dB_a_USA = sapply(results[[version]], function(x) round(x$dB_a[1], 4)),
    dB_a_China = sapply(results[[version]], function(x) round(x$dB_a[2], 4)),
    dB_a_ROW = sapply(results[[version]], function(x) round(x$dB_a[3], 4))
  )
  cat("\nSummary Table for", ifelse(version == "version1",
    "Version 1 (Tariffs as Taxes)", "Version 2 (Linear Tariff Scaling)"), ":\n")
  print(kable(summary_table, format = "markdown", align = "c"))
}
```