MAT 241 REVIEW SHEET FOR THE FINAL EXAMINATION
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SECTION ONE: EVALUATING LIMITS INCLUDING L‘HOPITAL‘S RULE.
Compute the following limits.
(i)\[ \lim_{x \to -1/2} \frac{2x^2 + 3x + 1}{4x^2 - 1} = \]?
(ii)\[ \lim_{x \to -3} \frac{5x + 15}{3x + 9} = \]?
(iii)\[ \lim_{x \to 1/3} \frac{4x^2 - 25}{9x^2 - 1} = \]?
(iv)\[ \lim_{x \to 0} \frac{\sin(x)}{\tan(x)} = \]?
(v)\[ \lim_{x \to -\infty} \frac{x + 2}{5x + 1} = \]?
(vi)\[ \lim_{x \to \infty} \frac{24x^4 + 9x^2 - x - 1}{10x^2 + 9x - 1} = \]?
(vii)\[ \lim_{x \to \infty} \frac{2}{e^{4x+5}} = \]?

SECTION TWO: DETERMINATION IF A FUNCTION IS CONTINUOUS
AND/OR DIFFERENTIABLE.
For each of the following functions, determine if each function is continuous and/or differentiable at the given point.
(i)\[ f(x) = \begin{cases} 
\ln(5x + 1), & x \geq 0 \\
2x + e^x, & x < 0. 
\end{cases} \]

SECTION THREE: DIFFERENTIATION OF FUNCTIONS.
For each of the following functions, find the derivative of each function.
(i)\[ G(x) = 8x^2 e^{9x-2}, \]
(ii)\[ H(x) = \frac{2x + 1}{x - 3}. \]
(iii)\[ I(x) = \ln(\ln(4x^2 + 9x - 1)). \]
(iv)\[ K(x) = 6\arcsin(x^2 + 1) - 2\csc(x^2 + 1). \]

SECTION FOUR: APPLICATIONS OF THE DERIVATIVE.
(i) Suppose that the velocity of an automobile starting from rest is given by \( v(t) = \frac{80t}{t + 5} \) (in feet/second). Find the rate of acceleration when \( t = 5 \) seconds and \( t = 60 \) seconds.

(ii) Apply the closed interval method to find the absolute minimum and maximum points for the function, \( f(x) = \ln(x^2 - 3) - x \), over the closed interval [2, 5].

(iii) Find \( y' \): \( 8x^2 + 3y^2 = 11 \).

(iv) Suppose that the displacement model for a free falling object is determined to be \( s(t) = 8100 - 16t^2 \). Find the instantaneous velocity at \( t = 2 \) seconds and at \( 5 \) seconds.

(v) Find the average velocity of the object travelling between 1 and 3 seconds.

(vi) A statistician employed with the XYZ Corporation has been asked to model the revenue and cost functions with respect to the total level of output of chips. The XYZ Corporation is a mid-sized company specializing in the production of graphic chip accelerators. The statistician has acquired data and fit the data accordingly to some mathematical behavior. The revenue and cost functions are:

\[
R(x) = x^3 + 2x^2 + 19x + 5 \quad \text{and} \quad C(x) = \frac{2}{3}x^3 + 6x^2 + 4x + 2.
\]

The chief financial officer has to address production concerns to the chief executive officer and they have to decide as to what will be the optimal level of output of chips. Based on the information given, answer the following questions.

(a) What is the profit function?

(b) Find all critical points for \( x \), the level of output of chips.

(c) Determine the relative extrema for the critical points. Also, determine how many graphic accelerators the company must sell so as to maximize its profits.

(d) What will the firm’s maximum profits?

(e) What price must XYZ charge to its customers in order to maximize profits?

(vii) Use derivatives to sketch the given function:

\[
f(x) = \frac{1}{4} x^4 - \frac{1}{2} x^2 + 1
\]

SECTION FIVE: INTRODUCTION TO INTEGRATION THEORY.

Find the anti-derivative for each of the following problems.

(i) \( \int \left( \frac{3}{x} + \frac{2}{x^2} - \frac{4}{x^3} + \frac{5}{x^4} \right) dx = ? \)

(ii) \( \int (3e^{8x-1} - \sec(x)\tan(x))dx = ? \)

(iii) \( \int (14e^{2x-1} + \sin(x) - 10\sec^2(x))dx \)

(iv) \( \int (16x^4 - 9x^3 + 14/x - 6/5x^7)dx \)

(v) \( \int (e^{12x+18})dx, \) from 0 to 1.